Implementation of an *a posteriori* error indicator for non-linear problems in soft-tissue biomechanics

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Error estimators for Finite Element (FE) solutions are well developed for linear problems [1]. They are very useful for determining the accuracy and the reliability of numerical solutions. For non-linear problems however, often encountered in soft-tissue biomechanics, very few investigations have been made and this issue is an ongoing subject of research [2].

In this contribution, we focus on the implementation of a recovery-based error indicator developed by [3] for linear problems. Our motivation is to extend this technique for non-linear problems.

The underlying idea of this technique is to compute the stress jump from one element to another. More precisely, given an enhanced stress field (calculated in this work as the linear interpolation of the stress) which takes the value $\bar{\tau}^*$at each node, it is possible to evaluate the FE error over each element $\|\varepsilon_\tau\|_e^2$ by equation (1) [1]

$$\|\varepsilon_\tau\|_e^2 = \int_{\Omega_e} (\bar{\tau}^* - \bar{\tau}_h).\,\mathbb{C}^{-1}.\,[\bar{\tau}^* - \bar{\tau}_h]\, d\Omega_e \qquad (1)$$

where *e* is an element of the FE mesh whose volume is Ω_e, $\bar{\tau}_h$ is the stress vector at each node computed from the integration points of *e* and $\mathbb{C}$ the tangent stiffness matrix.

The total error is then computed as the sum of the errors on each element of the mesh.

Given the difficulties in extracting $\mathbb{C}$ in standard commercial FE codes, we chose, as a first approximation, to replace it with the identity matrix $\mathbb{I}$. The rationale behind this hypothesis is that substituting $\mathbb{I}$ to $\mathbb{C}$ in equation (1) gives a measure of the stress jumps at the boundaries level. This jump is an indicator of the discretization error and therefore can be used as a basis for comparison of the accuracy between different meshes. More specifically, in that case, the product $(\bar{\tau}^* - \bar{\tau}_h).\,\mathbb{C}^{-1}.\,[\bar{\tau}^* - \bar{\tau}_h]$ can be simplified as a norm $\|\bar{\tau}^* - \bar{\tau}_h\|^2$ and equation (1) can be further simplified as

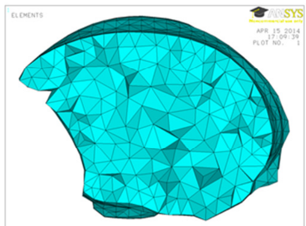
$$\|\epsilon_\tau\|_e^2 = \Omega_e.\,\|\bar{\tau}^* - \bar{\tau}_h\|^2 \qquad (2)$$

The technique is applied to assess the quality of different three-dimensional tongue meshes created as part of the ongoing work at our laboratory on the development of patient-specific biomechanical models to evaluate the consequences of intra-oral surgery on tongue mobility and speech production (figure 1). Beyond this specific example, we think these results could be applied to evaluate any FE model consisting of soft tissues.

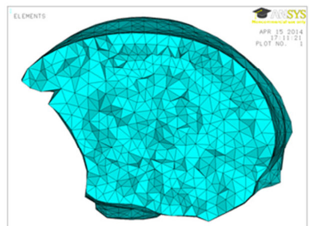
The next step to further develop the proposed tool will include $\mathbb{C}$ in the calculation of the *a posteriori* error estimator using the USERELEM capabilities of ANSYS with the aim of accounting for non-linear effects.

[1] P. Wriggers, « Adaptive Methods », in *Nonlinear Finite Element Methods*, Springer Berlin Heidelberg, 2008, p. 273‑311.
[2] R. Rannacher and J. Vihharev, « Adaptive finite element analysis of nonlinear problems: balancing of discretization and iteration errors », *Journal of Numerical Mathematics*, vol. 21, n° 1, janv. 2013.

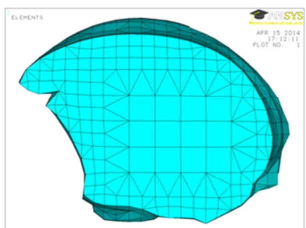
[3] O. C. Zienkiewicz and J. Z. Zhu, « The superconvergent patch recovery (SPR) and adaptive finite element refinement », *Computer Methods in Applied Mechanics and Engineering*, vol. 101, n° 1‑3, p. 207‑224, dec. 1992.



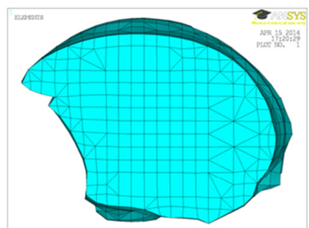
(a)
Tetrahedral mesh (coarse)



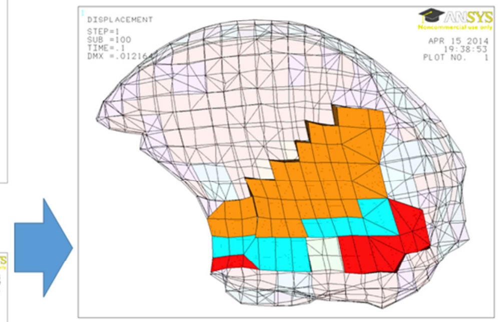
(b)
Tetrahedral mesh (Fine)



(c)
Mixed tetrahedral/
hexahedral mesh (coarse)



(d)
Mixed tetrahedral/
hexahedral mesh (Fine)



(e)
Example of deformed mesh under muscle activation of internal tongue muscle.

The error indicator implemented is used to assess the relative error of the finite element solution obtained using the different three-dimensional meshes.